SIMULTANEOUS MEASUREMENT OF THE STAGNATION TEMPERATURE AND SPECIFIC MASS FLOW RATE IN A HYPERSONIC GAS FLOW

G. E. Pervushin

UDC 533.6.071.082.61

The procedure and results of simultaneous measurement of the stagnation temperature and specific mass flow of gas in a hypersonic flow by means of heated-chamber impact tubes is discussed. The construction of the tubes is described.

In the investigation of hypersonic flows it is necessary to determine the velocity, density, and temperature or enthalpy of the gas. For this purpose two or three independent measurements are made with different impact tubes. Nikol'skii [1] has consolidated measurements of the total head and heat flux to a hot-wire probe and from the results determined the specific mass flow of gas (i.e., the product of the density and velocity of the gas).

For the investigation of hypersonic gas flows in intermittent (gun-type) facilities, difficulties arise insofar as it is impossible to perform measurements at one point of the flow with different tubes during one shot, while measurements performed in different shots incur an additional error associated with scatter of the values of the flow parameters. Moreover, measurements of the temperature of a gas flow by measn of stagnation-chamber tubes are accompanied by the additional difficulty that there is not sufficient time for the thermocouple junction and chamber walls to be heated to a steady-state temperature.

Accordingly, the conventional tubes that have been used successfully to measure the stagnation temperature of longer-acting (quasicontinuous) facilities [2] are inapplicable in intermittent facilities. However, if a changeover is made from the steady-state to the regular measurement mode, it is no longer necessary for the tube to be heated by the gas flow to a steady-state temperature.

The regular measurement mode with simultaneous steady-state heating of the sensing element has been used previously [3-5]. It entails heating the tube to a definite temperature under steady-state conditions in vacuum. This process can be described by the heat-balance equation formulated for the sensing element (thermocouple junction or working section of a resistance thermometer): $Q = Q_\varepsilon + Q_\lambda$.

After the sensing element has been heated to a certain steady-state temperature T, the tube is inserted into the investigated flow or the test facility is fired. The sensing element will be heated if $T < T_0$, or it will be cooled if $T > T_0$. This process is described by the heat-balance equation

$$Q \pm hS(T_0 - T) = Q_\varepsilon + Q_\lambda \pm \frac{c\gamma b}{2}\frac{\Delta T}{\Delta\tau} + \Delta Q_\varepsilon + \Delta Q_\lambda,$$

from which we infer

$$\pm Sh(T_0 - T) = \mp \frac{c\gamma b}{2}\frac{\Delta T}{\Delta\tau} + \Delta Q_\varepsilon + \Delta Q_\lambda. \tag{1}$$

The thickness of the sensing element can be chosen in such a way as to minimize the radiant heat losses during the measurement period in comparison with the heat flux accumulated by the element:

$$\Delta Q_\varepsilon \simeq 4\varepsilon\sigma T^3 \Delta T \ll \frac{c\gamma b}{2}\frac{\Delta T}{\Delta\tau}.$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 43, No. 5, pp. 759-766, November, 1982. Original article submitted August 25, 1981.

For a given measurement error δ this relation can be used to estimate the smallest allowable thickness of the element:

$$b \geqslant \frac{8\varepsilon\sigma T^3 \Delta\tau}{\delta c\gamma}. \tag{2}$$

Moreover, the variation of the heat losses through the leads can be estimated from the solution of the one-dimensional heating problem with heat input from the sidewalls of a rod of length l under the following conditions: a) At the attachment sites of the leads in the ceramic the temperature remains constant during the measurement period [$T(0;\ \tau)$ = const, i.e., the ceramic has a high heat capacity]; b) the sensing element and leads are heated (or cooled) over their entire length by a uniform specific heat flux, equal to $q = c\gamma b\Delta T/2\Delta\tau$. The convective heat flux to these objects is $Q_c = c\gamma b\Delta TS/2\Delta\tau = \pi c\gamma bDl\Delta T/\Delta\tau$.

Instead of estimating the variation of the heat flux by heat conduction, it is more practical to consider the heating problem for a rod thermally insulated at the sidewalls and heated at one end at a constant rate ($\Delta T/\Delta\tau = k$). The initial temperature is uniform along the rod. The rod is thermostatically regulated at the other end:

$$T(x;\ 0) = T(0;\ \tau) = 0,\ T(l;\ \tau) = k\tau;\ \frac{\lambda}{c\gamma}\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial\tau}.$$

The solution of this problem can be written in the form

$$T(x;\ \tau) = \frac{-kx}{l} + \frac{k}{l}\sum_{n=1}^{\infty}\frac{(-1)^n\, 2l^3 c\gamma}{\pi^3 n^3\lambda}\left[1 - \exp\left(-\frac{\pi^2 n^2\lambda\tau}{c\gamma l^2}\right)\right]\sin n\pi\frac{x}{l},$$

whereupon

$$\delta = \frac{\Delta Q_\lambda}{Q_c} = \frac{-\lambda\dfrac{\partial T(l;\ \tau)}{\partial x}S'}{\dfrac{\pi c\gamma bl D\Delta T}{\Delta\tau}} = \frac{\lambda\tau D}{l^2 c\gamma b}.$$

From this relation we can estimate the shortest length of the leads and, hence, the length of the tube chamber. For a given length as well as given values of the time and error of measurement, the variation of the heat losses by conduction can be neglected in comparison with the convective heat flux from the gas:

$$l^2 \geqslant \lambda\tau/c\gamma b\delta. \tag{3}$$

Under conditions (2) and (3) Eq. (1) acquires the form

$$h(T_0 - T) = \frac{c\gamma b}{2}\frac{\Delta T}{\Delta\tau}.$$

As $\Delta T \to 0$ the coefficient $h(T_0-T) \to 0$ and so $T \to T_0$. However, it is practically impossible to heat the sensing element to the temperature T_0, because T_0 is not known prior to the experiment. The measurements were therefore performed several times at various initial temperatures T of the sensing element, which were established by varying the heater power input. For $T > T_0$ the ratio $\Delta T/\Delta\tau < 0$, and for $T < T_0$ we have $\Delta T/\Delta\tau > 0$.

The initial temperature T of the sensing element was measured prior to each shot of the test facility, and during its operation (i.e., during the period in which the gas flow acts on the sensing element), its heating (or cooling) rate $\Delta T/\Delta\tau$ was measured.

Regarding $\Delta T/\Delta\tau$ as a function of the initial temperature, the temperature T_x at which $\Delta T/\Delta\tau = 0$ is determined by interpolation. In this case $\Delta T = 0$, and so $h(T_0-T_x) = 0$, whereupon $T_x = T_0$.

In the foregoing heat-balance equations it has been assumed that the rate of change of the temperature $\Delta T/\Delta\tau$ along the thickness of the sensing element (thermocouple junction) is uniform.

The time required for equalization of the heating rates along the thickness can be estimated from the solution of the heat-conduction equation for an infinite plate:

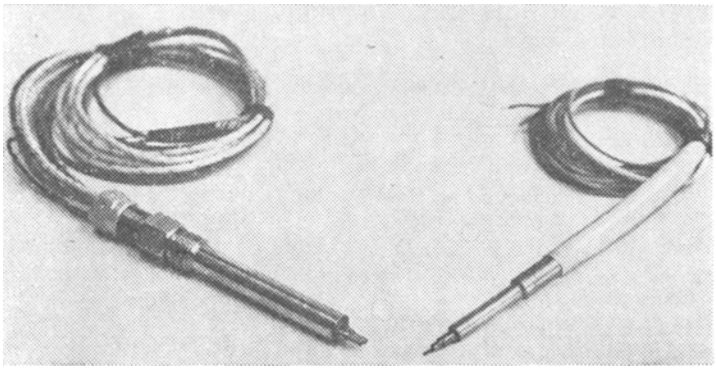


Fig. 1. Impact tubes.

$$T(x;\ 0)=0;\quad \frac{\partial T(b/2;\ \tau)}{\partial x}=0;\quad -\lambda\frac{\partial T(0;\ \tau)}{\partial x}=q;$$

$$\frac{\lambda}{c\gamma}\ \frac{\partial^2 T(x;\ \tau)}{\partial x^2}=\frac{\partial T(x;\ \tau)}{\partial \tau};$$

x = 0 at the surface of the sensing element.

The solution of this problem has the form [6]

$$T(x;\ \tau)=\frac{q}{\lambda}\left[\frac{2\lambda\tau}{c\gamma b}-\frac{b^2-12x^2}{12b}+\frac{b}{2}\sum_{n=1}^{\infty}\frac{(-1)^n 2}{\pi^2 n^2}\exp\left(-\frac{\pi^2 n^2\lambda\tau}{c\gamma b^2}\right)\cos\frac{2x}{b}n\pi\right],$$

whence we can determine the rates of change of the temperature of the plate and its median plane:

$$\frac{\partial T(0;\ \tau)}{\partial \tau}=\frac{q}{\lambda}\left[\frac{2\lambda}{c\gamma b}+\frac{b}{2}\sum_{n=1}^{\infty}\frac{(-1)^n 8\lambda}{c\gamma b^2}\exp\left(-\frac{4\pi^2 n^2\lambda\tau}{c\gamma b^2}\right)\right],$$

$$\frac{\partial T(b/2;\ \tau)}{\partial \tau}=\frac{q}{\lambda}\left[\frac{2\lambda}{c\gamma b}\right].$$

The equalization time of the heating rates is determined from the condition that their relative difference should not be greater than the allowable error of measurement (δ). If we assume that the allowable error is not to exceed 3%, then

$$\delta=\frac{\dfrac{\partial T(0;\ \tau)}{\partial \tau}-\dfrac{\partial T(b/2;\ \tau)}{\partial \tau}}{\dfrac{\partial T(b/2;\ \tau)}{\partial \tau}}\simeq 2\exp\left(-\frac{4\pi^2\lambda\tau}{c\gamma b^2}\right).$$

Accordingly, for a Chromel–Alumel thermocouple with c = 460 J/kg, γ = 8000 kg/m^3, λ = 22.6 J/m·sec·°C, and b = $3\cdot 10^{-5}$ m we obtain the following time of equalization of the heating rates along the thickness of the junction, providing an estimate of the response time of the tube:

$$\tau_0 \geqslant -\frac{c\gamma b}{4\pi^2\lambda}\ln\frac{\delta}{2}\approx 10^{-5}\ \text{sec.} \tag{4}$$

This estimate of the response time has the advantage that it does not depend on the value of the heat flux (i.e., on the magnitude of the input effect), but is governed entirely by the thermophysical properties of the sensing element and the allowable measurement error δ. Thus defined, the response time of the tube will be identical for operation of the tube in different media and with different flow rates of the medium or velocities of the tube. This definition is still not universal, because τ_0 = const under the condition $\lambda\dfrac{\partial T(0;\ \tau)}{\partial x}=q=\text{const}$.

It is essential to note that, as a rule, the relative error of measurement of T_0 is much smaller than δ, because the initial temperatures T are measured with high accuracy under steady-state conditions prior to firing of the tunnel, and δ governs the correction error, which is determined from oscillograms.

For a given measurement time τ_0 and error δ, relation (4) can be used to estimate the maximum thickness of the sensing element.

To implement the method we used a tube whose chamber was made of Nichrome in the form of a cylinder with two notches cut along its generatrices. The chamber was heated by a current flowing along both sides of the cylinder (Fig. 1). The junction of a Chromel—Alumel thermocouple, serving as the sensing element, was installed at the forward orifice. The entrant opening of the chamber had a diameter of 0.8 mm, and the outside diameter was 1.2 mm.

During the experiment the tube was placed so that the axis of its chamber was directed along the streamlines. The heater chamber was connected to a power supply. The desired initial temperature of the sensing element was set by varying the heating current and was monitored with a PP63 potentiometer.

During a shot (when the gas flow as acting on the tube) the temperature variation of the sensing element was recorded by means of an amplifier and an N-115 oscillograph, which incorporates M004-7,00 moving-coil elements with a natural frequency of 7 kHz, so that the actual resolving time is less than or equal to 200 μsec. The problem of determining the temperature of the gas flow from these measurements is equivalent to the reverse interpolation problem for the function $\Delta T/\Delta\tau = f(T)$, i.e., to determine the value of T_X such that $\Delta T/\Delta\tau = f(T_X) = 0$. Inasmuch as the heat-transfer coefficient is positive and nonzero for any values of T and T_0, the function $f(T)$ is monotonic and single-valued for any T, so that the problem of determining T_0 must have a unique solution.

The heat-transfer coefficient is known to depend only very slightly on the temperature difference T_0—T. For example, in the Fay—Riddell equation [7] the heat-transfer coefficient depends on the temperature factor T/T_0 to the power 0.1, so that in measurements with small differences T_0—T the heat-transfer coefficient can be assumed constant, and the function $\Delta T/\Delta\tau = f(T)$ linear.

From the results of measurements carried out for various freestream parameters and for temperature differences T_0—T not exceeding (in absolute value) 100°C we have plotted the linear regressions $\Delta T/\Delta\tau = aT + b$. In every case the correlation coefficients were greater than 0.97.

Consequently, in order to determine the stagnation temperature of a gas flow it is sufficient to perform measurements for two different initial temperatures of the sensing element or with two probes during one shot of the test facility, where the sensing elements of the probes have different temperatures T' and T" prior to the shot (or prior to insertion of the tube into the flow). The temperatures of the junctions will change after the test facility is fired, and the heat-balance equations describing this process have the form

$$\frac{c_1\gamma_1 b_1}{2}\,\frac{\Delta T'}{\Delta\tau} = h\,(T_0 - T');\quad \frac{c_2\gamma_2 b_2}{2}\,\frac{\Delta T''}{\Delta\tau} = h\,(T_0 - T''),$$

so that

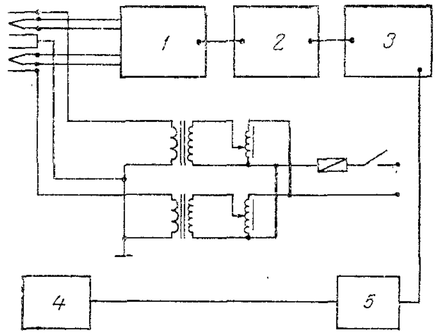
$$T_0 = \left(\frac{c_1\gamma_1 b_1\Delta T'/\Delta\tau}{c_2\gamma_2 b_2\Delta T''/\Delta\tau}\,T'' - T'\right)\Big/\left(\frac{c_1\gamma_1 b_1\Delta T'/\Delta\tau}{c_2\gamma_2 b_2\Delta T''/\Delta\tau} - 1\right). \tag{5}$$

Figure 2 shows a block diagram of the arrangement for measuring intermittent gas flows with a tube having two heated chambers.

The ratios $c_1\gamma_1 b_1/c_2\gamma_2 b_2$ can be determined by writing the rates of change of the junction temperatures in the gas flow without heating of the chamber. In this case the initial temperatures of the sensing elements are identical, and from the heat-balance equations (5) for the heating process we obtain

$$c_1\gamma_1 b_1/c_2\gamma_2 b_2 = (\Delta T'/\Delta\tau)/(\Delta T''/\Delta\tau).$$

If τ_0 [given by relation (4)] is one-tenth the measurement time, which is determined by the lifetime of the investigated gas flow, the temperatures T' and T" and their variations $\Delta T'/\Delta\tau$ and $\Delta T''/\Delta\tau$ must be regarded as time functions, from which the stagnation temperature of the gas flow can be determined as a function of the time.



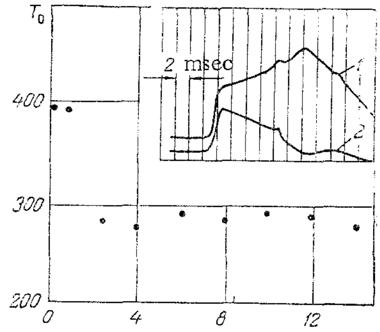


Fig. 2 Fig. 3

Fig. 2. Block diagram of the measurement arrangement. 1) Unit for emf compensation; 2) amplifier; 3) N-115 oscillograph; 4) synchronization sensor; 5) synchronizer.

Fig. 3. Oscillograms of sensing-element temperatures for initial values T' = 250°C (trace 1) and T" = 325°C (trace 2) and graph of stagnation temperature (points): T_0, °C, vs τ, msec.

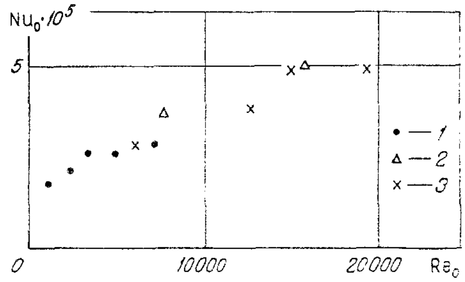


Fig. 4. Measurement results for: 1) M = 8; 2) 6; 3) 5.

Figure 3 shows oscillograms of T' and T" obtained in the measurement of T_0 with a two-chamber tube in a shock-driven (gun-type) wind tunnel operating in the Ludwieg configuration [8], with the gas in the fore chamber heated to 330°C. Air was evacuated from the working section and nozzle, which were separated from the fore chamber. The tunnel was fired at the instant of bursting of the diaphragm, whereupon a rarefaction wave propagated in the fore chamber, followed by a reduction in the temperature of the gas. Also shown in Fig. 3 are the results of calculating T_0 by graphical differentiation of these oscillograms. It was determined from the measurement results that for the first 1.5 msec the gas flows with a higher stagnation temperature than during quasisteady flow (up to 14 msec). The gas with the elevated temperature is clearly produced by the compression of the residual air in the working section and nozzle during firing of the tunnel.

From Eqs. (5), along with the stagnation temperature of the gas flow, we can also determine the heat-transfer coefficient $h = [c_2\gamma_2 b_2 \Delta T''/\Delta\tau - c_1\gamma_1 b_1 \Delta T'/\Delta\tau]/[2(T''-T')]$. In the probes used for these experiments the sensing element was a thermocouple junction situated on the axis of the chamber at its orifice toward the impingent gas flow. Assuming that the gas flow in the chamber and at its orifice is laminar, we can write a relation between the specific mass flow (product of the density ρ and the velocity u) of the gas and the heat-transfer coefficient, making use of the Reynolds analogy [9]:

$$Nu_0 = f(Re_0;\ Pr_0), \tag{6}$$

where $Nu_0 = hL/\lambda_0$ is the Nusselt number, L = const for each sensing element and in the present work is equal to the orifice diameter of the tube chamber (L = d), $Re_0 = \rho uL/\mu_0$ is the Reynolds number, and $Pr_0 = \mu_0 c_p/\lambda_0$ is the Prandtl number. Inasmuch as μ_0 and λ_0 are evaluated from the measured temperature T_0 and as the quantities L and c_p are constants, relation (6) can be used to determine the specific mass flow and to rewrite $Nu_0 = \varphi(\rho u)$. This function is usually written in the form $Nu_0 = \psi(Re_0)$. The relationship between the specific mass flow at the orifice of the tube chamber on its axis (ρu) on the specific mass flow in the impingent gas flow (free stream value $\rho_\infty u_\infty$) can be obtained from the equation of continuity $\partial\rho/\partial\tau$ + div $(\rho u) = 0$ written for the central gas streamer flowing around the thermocouple junction. If the measurement time $\tau \gg L/a_\infty$, the gas flow in the tube chambers can be regarded as steady, so that $\partial\rho/\partial\tau = 0$. Moreover, in transmission across the normal shock the direction of the gas particle velocity and the central streamer of the flow remain unchanged; it can therefore be assumed that $\partial(\rho u)/\partial y = \partial(\rho u)/\partial z = 0$. From this result we infer that $\partial\rho u/\partial x = 0$ and so on $\rho u = \rho_\infty u_\infty$, i.e., the specific mass flow in the tube chambers is determined by the specific free-stream mass flow and $Nu_0 = \psi(\rho_\infty u_\infty L/\mu_0)$. The form of this function in hypersonic flows does not depend on the Mach number M of the flow ($M = u_\infty/a_\infty$).

To calibrate the tube, i.e., to plot the function $Nu_0 = f(Re_0)$, we performed experiments in intermittent facilities at numbers M = 5, 6, 8. The tube was placed in the working section of the wind tunnel at the nozzle orifice. The numbers Re_0 were varied by varying the pressure in the fore chamber and the temperature T_0. The results of the measurements are shown in Fig. 4.

NOTATION

Q, heat input per unit time from heater to sensing element of impact tube; Q_ε, Q_λ, quantities of heat transferred from sensing element by radiation and conduction in loads; T, steady-state temperature of sensing element in heating before firing of wind tunnel; T_0, stagnation temperature of gas flow; S, surface area of sensing element; c, γ, b, specific heat, density, and thickness of sensing element; ΔT, temperature variation of sensing element; τ, time; ΔQ_ε, ΔQ_λ, variations of heat losses by radiation and conduction in leads from sensing element during measurement period; h, heat-transfer coefficient; ε, emissivity of surface of sensing element; σ, Boltzmann constant; δ, measurement error; l, length of leads; D, lead diameter; S', cross section of leads; λ, thermal conductivity; q, convective heat flux; λ_0, thermal conductivity of gas at temperature T_0; μ_0, viscosity of gas at temperature T_0; c_p, specific heat of gas at constant pressure; ρ_∞, u_∞, freestream values of density and velocity of gas; M, freestream Mach number; a_∞, speed of sound.

LITERATURE CITED

1. Yu. V. Nikol'skii, "Experimental study of the flow field on the leeward side of a plate in hypersonic flow," Uch. Zap. Tsentr. Aerogidrodin. Inst., 9, No. 2, 19-26 (1978).
2. A. N. Petunin, Techniques for Measuring the Parameters of a Gas Flow [in Russian], Mashinostroenie, Moscow (1972).
3. I. E. Laggraff, The Design Instrumentation and Calibration of the Oxford University Hypersonic Gun Tunnel, Rep. No. 1094, Dept. Eng. Sci., Oxford Univ. (December, 1969).
4. E. I. Soffly, Use of Pulse-Heated Fine Wire Probe for Measurement of Total Temperature in Shock-Driven Facilities, AIAA Paper No. 68-393 (1968).
5. B. E. Zhestkov and A. V. Lipin, Inventors' Certificate No. 329412 (USSR): "Sensor for measuring the temperature of a gas flow," Byull. Izobret., No. 17 (1972).
6. A. V. Lykov, Theory of Heat Conduction [in Russian], Vysshaya Shkola, Moscow (1967).
7. D. A. Fay and F. R. Riddell, "Theory of stagnation point heat transfer in dissociated air," J. Aeronaut. Sci., 25, No. 2, 74-96 (1958).
8. H. Ludwieg, T. Hattner, and H. Grane-Carstensen, "Der Rohrwindkanal der aerodynamischen Versuchsanstalt," in: Jahrbuch 1969, DGLR Braunschweig, Göttingen (1970), pp. 52-58.
9. H. Schlichting, Boundary Layer Theory (6th ed.), McGraw-Hill, New York (1968).